Domain Decomposition and Parallel Direct Solvers as an Adaptive Multiscale Strategy for Damage Simulation in Materials

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Goal

Numerical model of damage in heterogeneous materials
→ fracturing in concrete

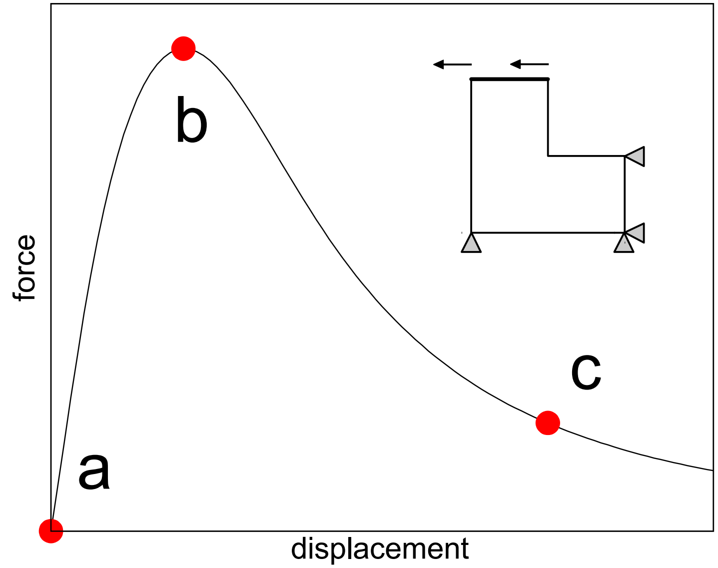
- Simulate material damage
  - Realistic
  - Efficient
  - Robust
- Damage model properties
  - Mesh objectivity
  - Usable in standard FEM

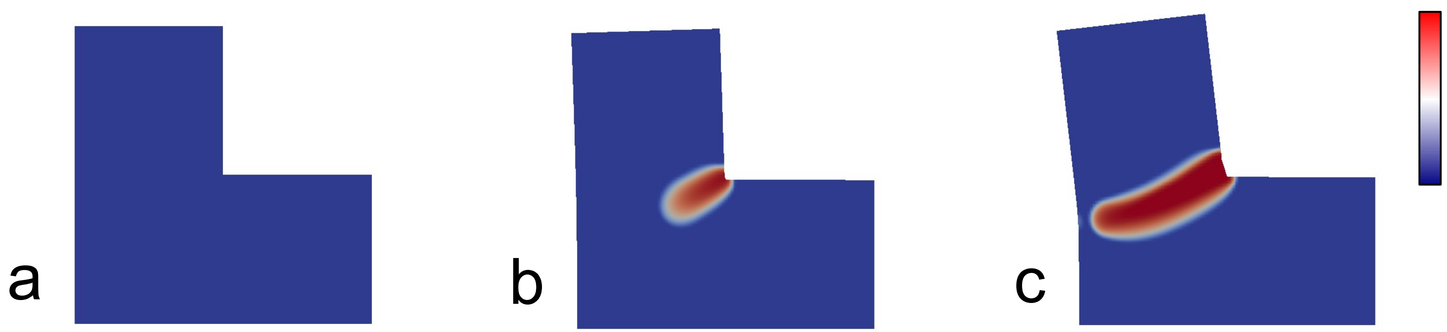
Gradient Enhanced Damage Model

Peerlings et al. (1995)

- Extra dof, drives damage
- Damage scalar ω:

$$D \rightarrow D(1 - \omega)$$





Challenges

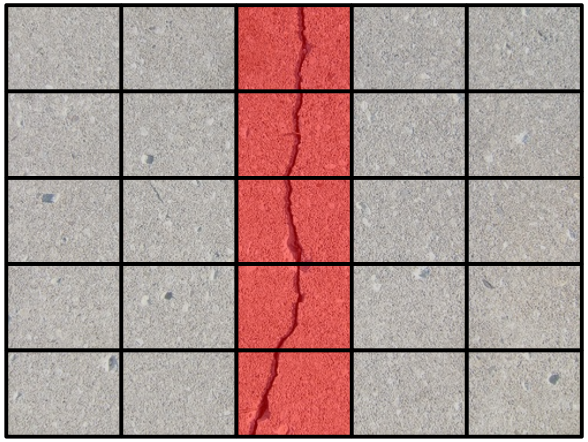
- Highly non-linear: requires iterations within increments
- Stiffness matrix is asymmetric
  - Solvers:
    - Umfpack (serial)
    - Pardiso, PaStiX, MUMPS (parallel)
- Crack localization necessitates fine mesh

$\rightarrow$ Full numerical solution is expensive

Proposed framework

Multiscale Domain Decomposition

- Subdivide into domains
  → Only part of material needs to have detailed mesh
- Start out with all coarse mesh
- Perform mesh refinement, driven by criteria coupled with damage parameter
- Solver available → FETI



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FETI (classic)

- Decomposition:
  - Lagrange multipliers for boundary conditions

    Singular Stiffness matrix
  - Non-conforming meshes → LMP constraints

$$K \cdot u = f$$

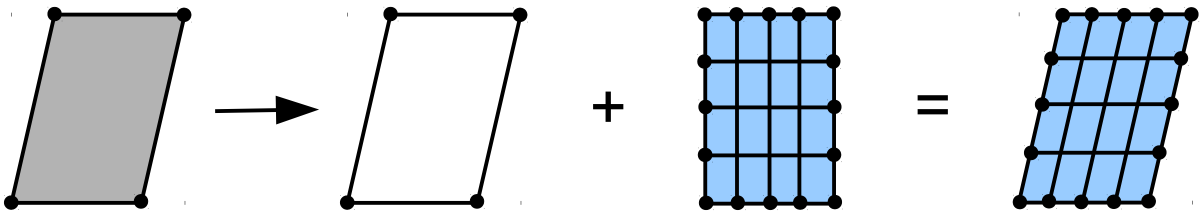
$$\downarrow$$

$$\begin{bmatrix} K & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$

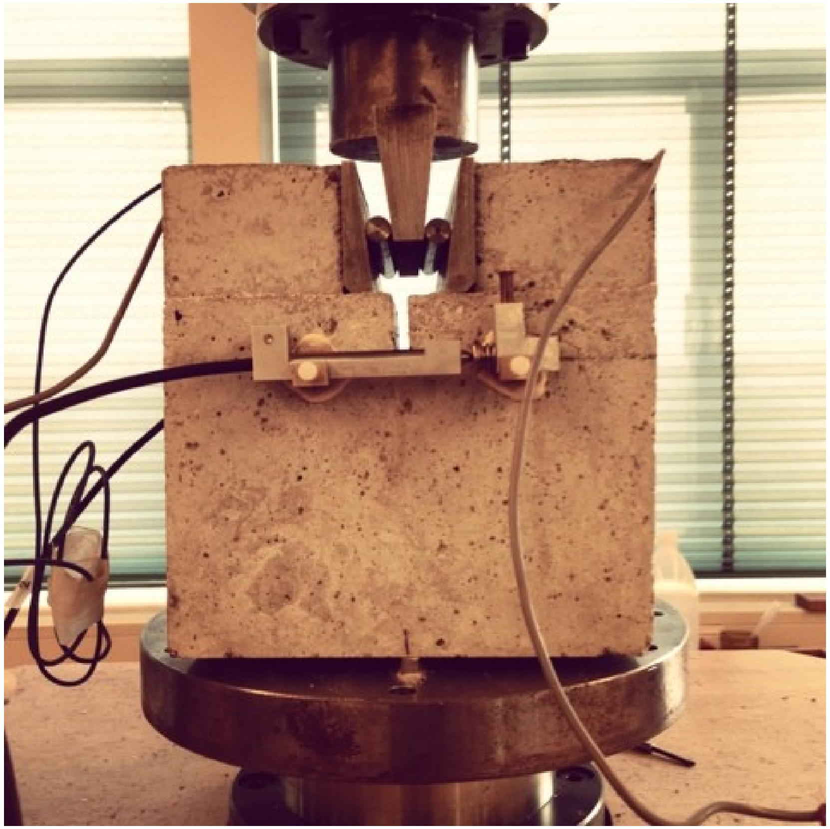
- Solver for Interface problem:
  - (Direct)
  - Iterative → BiCGStab
    - Preconditioning (SuperLumped, Lumped, Dirichlet)
    - Scaling (m,k,s)

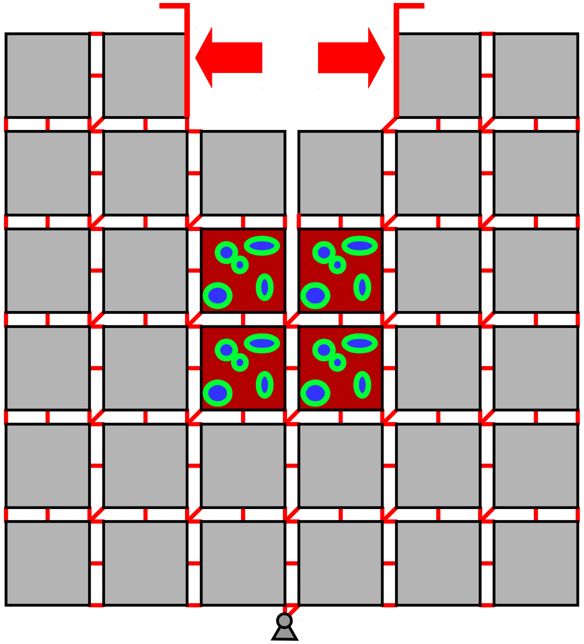
Multiscale → Domain Refinement

- Decide when and where to refine
  - Depends on model
  - Heuristic → User defined criteria
- Solve Boundary Value Problem



Model Wedge Split test

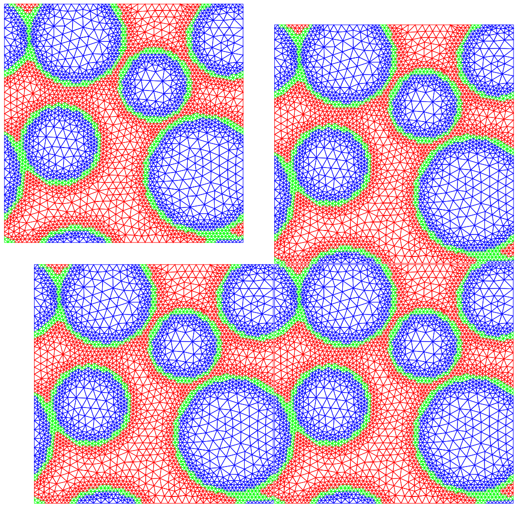




RVE and Domain

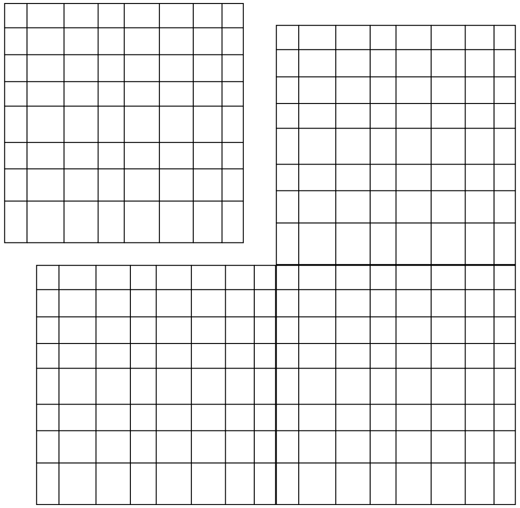
Meso (“fine”)

- Heterogeneous
- Matrix | ITZ | Aggregate
- T3 1ip 11747n 23112e



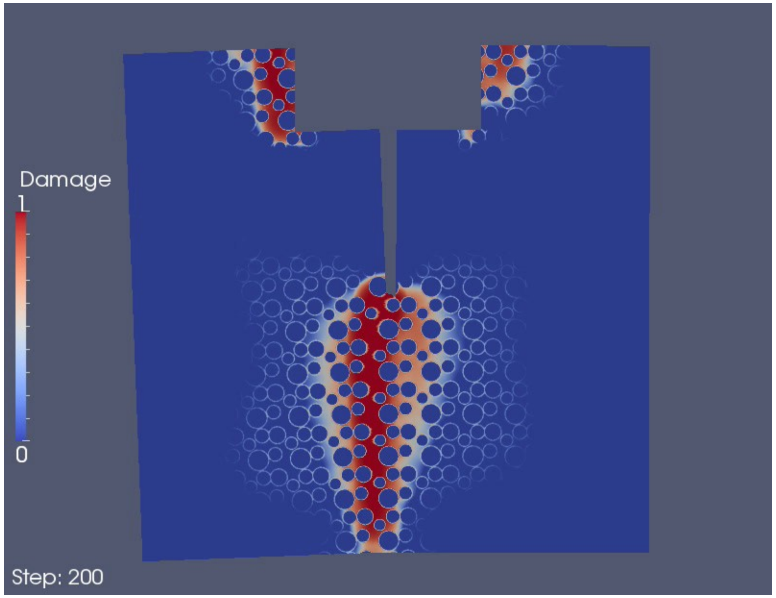
Macro (“coarse”)

- Homogeneous
- D_{eff}
- Q4 4ip 289n 256e

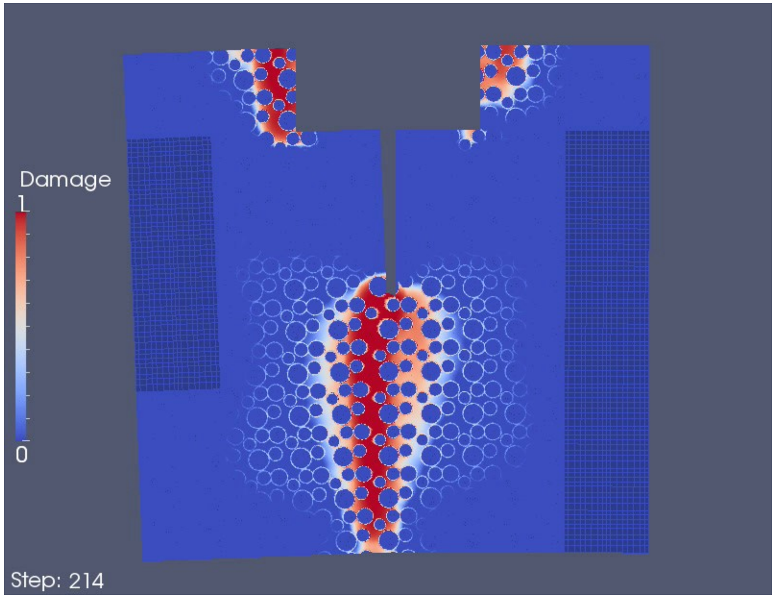


Numerical Results

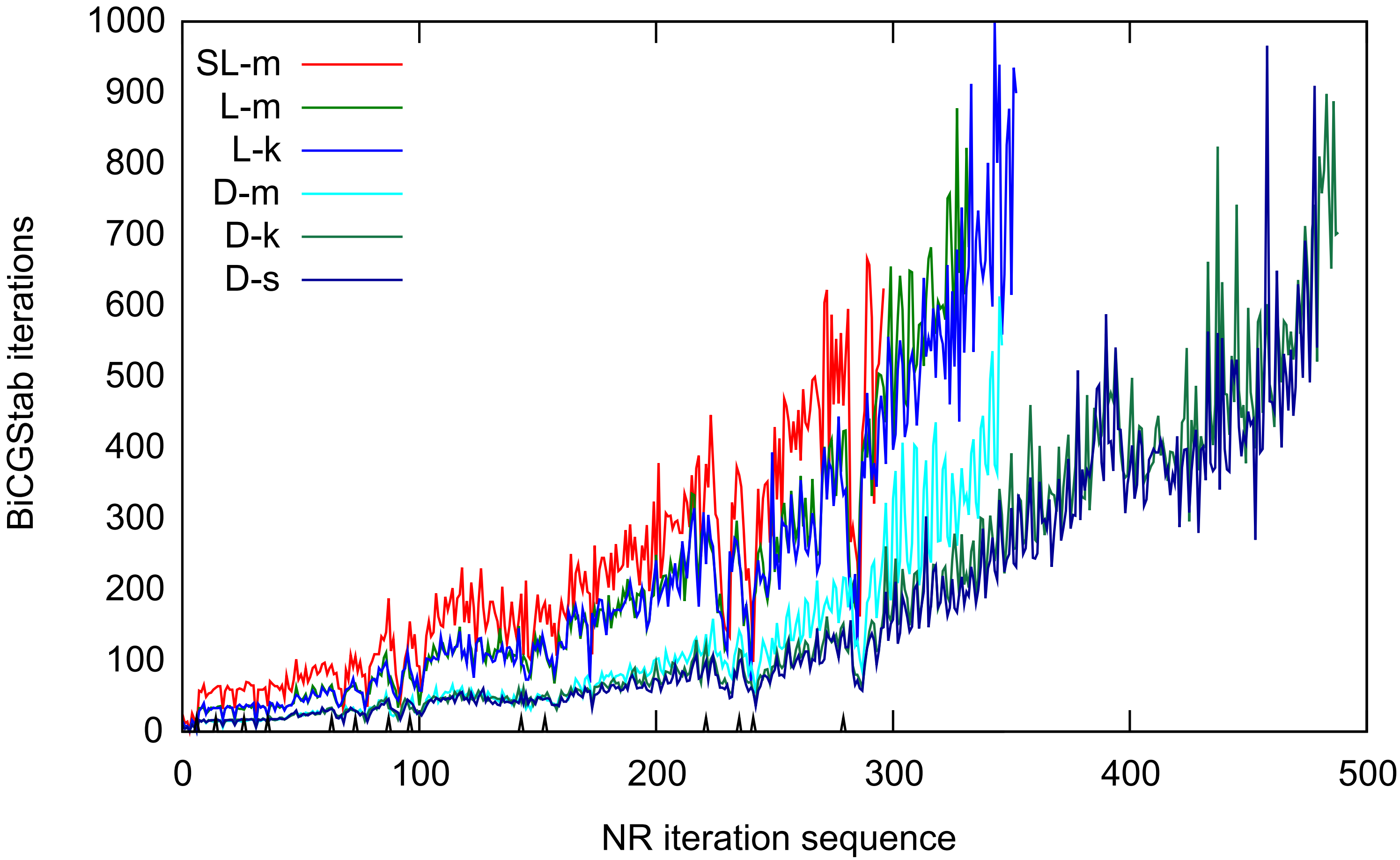
- FNS (Umfpack: 24.5 hours)



- DD 34 (FETI Direct: 59.2 hours)



FETI iterative solver



Problem

- BiCGStab solver takes many iterations to converge

  → Standard preconditioners are not good enough

- Possible causes:
  - Stiffness scaling (k/s) on non-stiffness components

$$\begin{bmatrix} K^{uu} & K^{ue} \\ K^{eu} & K^{ee} \end{bmatrix} \cdot \begin{bmatrix} \delta u \\ \delta u_e \end{bmatrix} = RHS$$

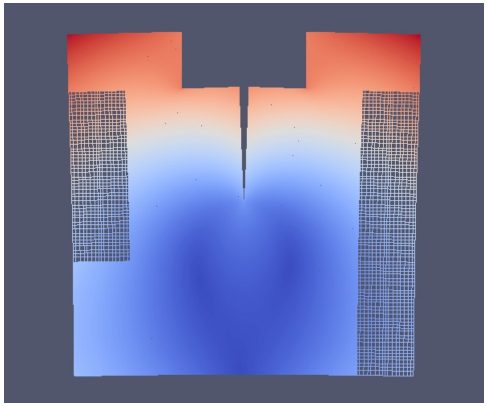
  - Damage induced heterogeneity

Damage induced Heterogeneity

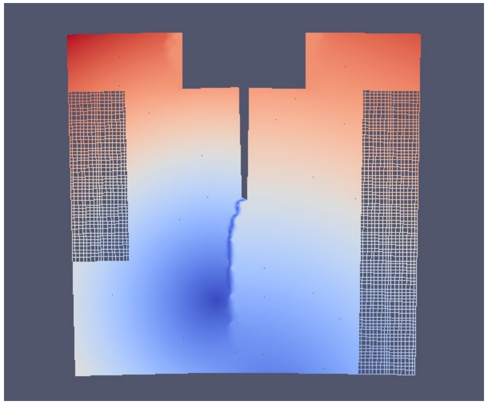
- Linear elastic test on final refined mesh + iterative solver
  - Replace ***D*** → ***D****(1 – ω)*
  - Choose different damage profiles

	First Iter L-k	First Iter D-k
ω_0	39	16
ω_{100}	781	233
ω_{214}	>5000	1936

Damage induced heterogeneity indeed causes poor convergence



ω_0



ω_{100}

Possible Solution

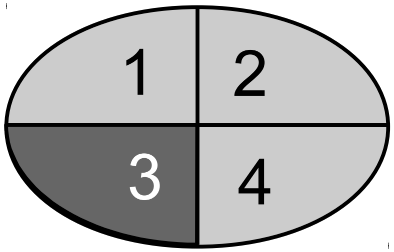
- Eigenvalue methods:
  - local generalized eigenvalue problems
  - FETI-GenEO (Spillane & Rixen, 2013)
    → some adaptation needed

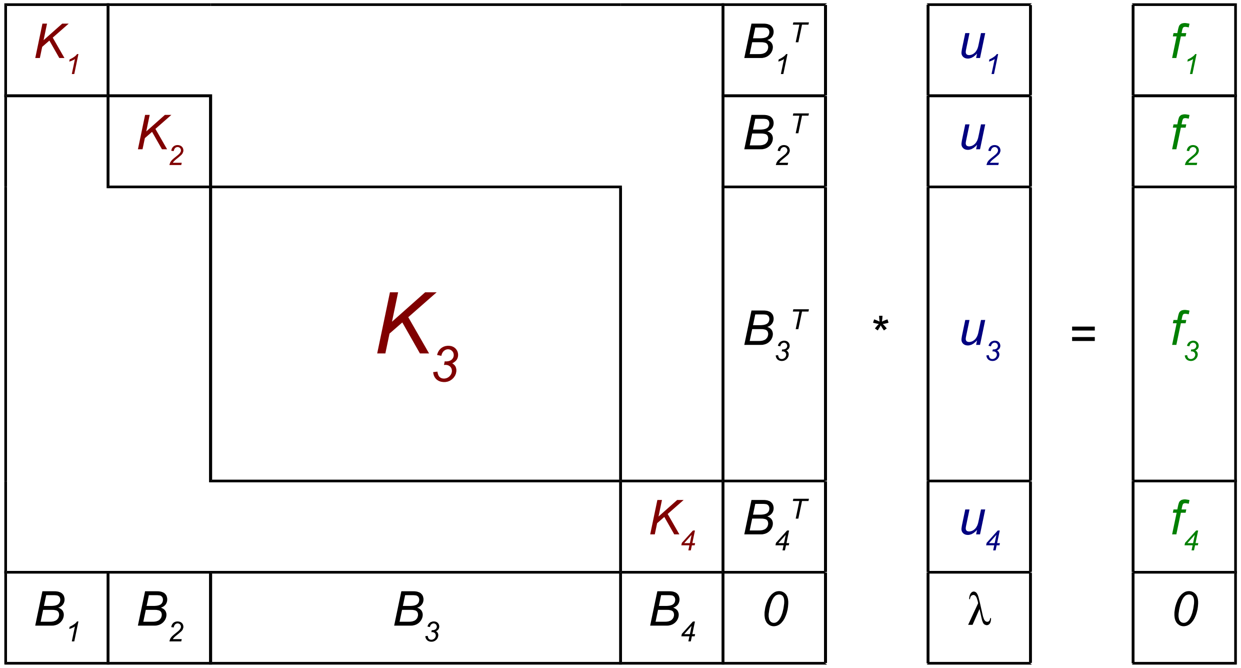
- Idea: Can we combine direct solver robustness with multiscale decomposition?

  Yes, via the full Dual assembly

Multiscale → Full Dual assembly

$$\begin{bmatrix} K & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} u \\ \lambda \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix}$$





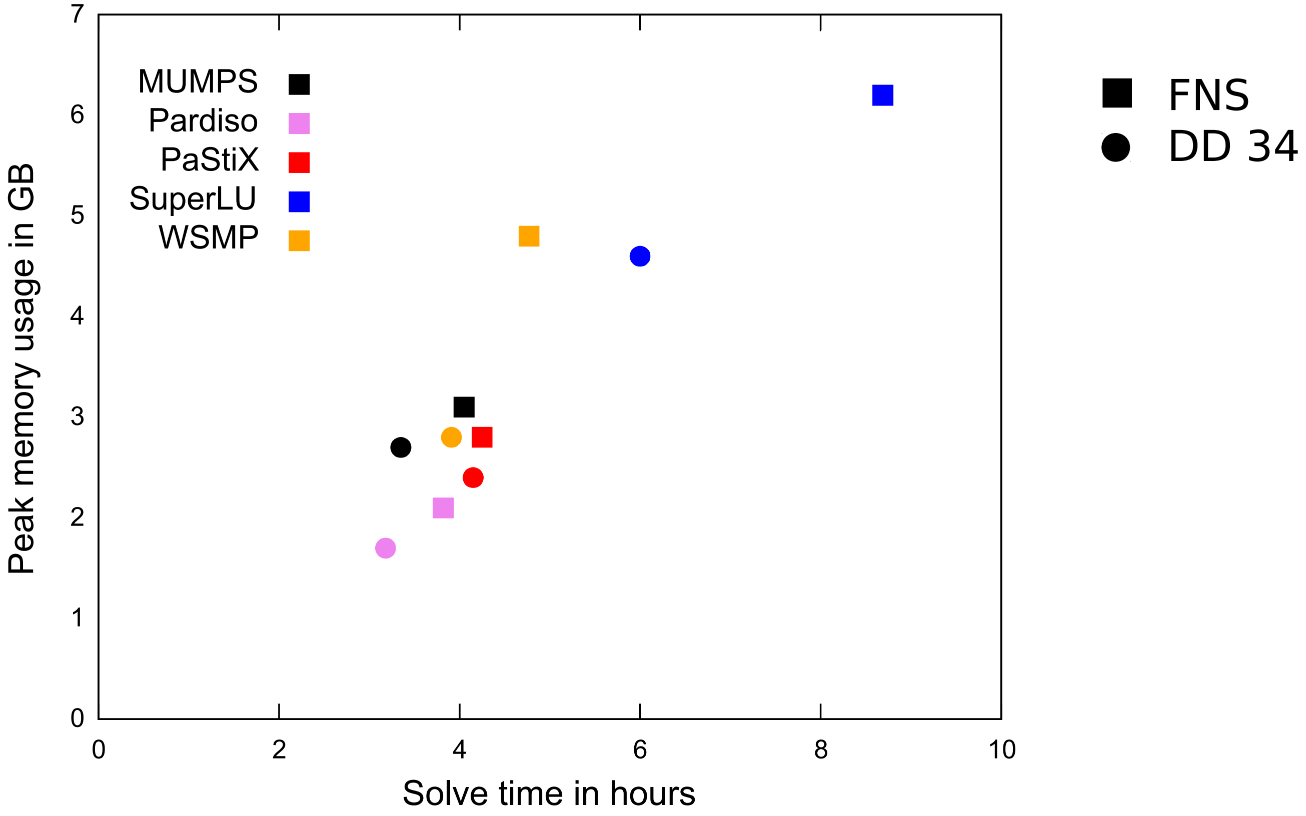
Parallel direct solver + Multiscale + Full Dual assembly

- Less dof's than FNS
  - Majority of domains are coarse “macro” domains
  - Domain refinement induced only by damage growth

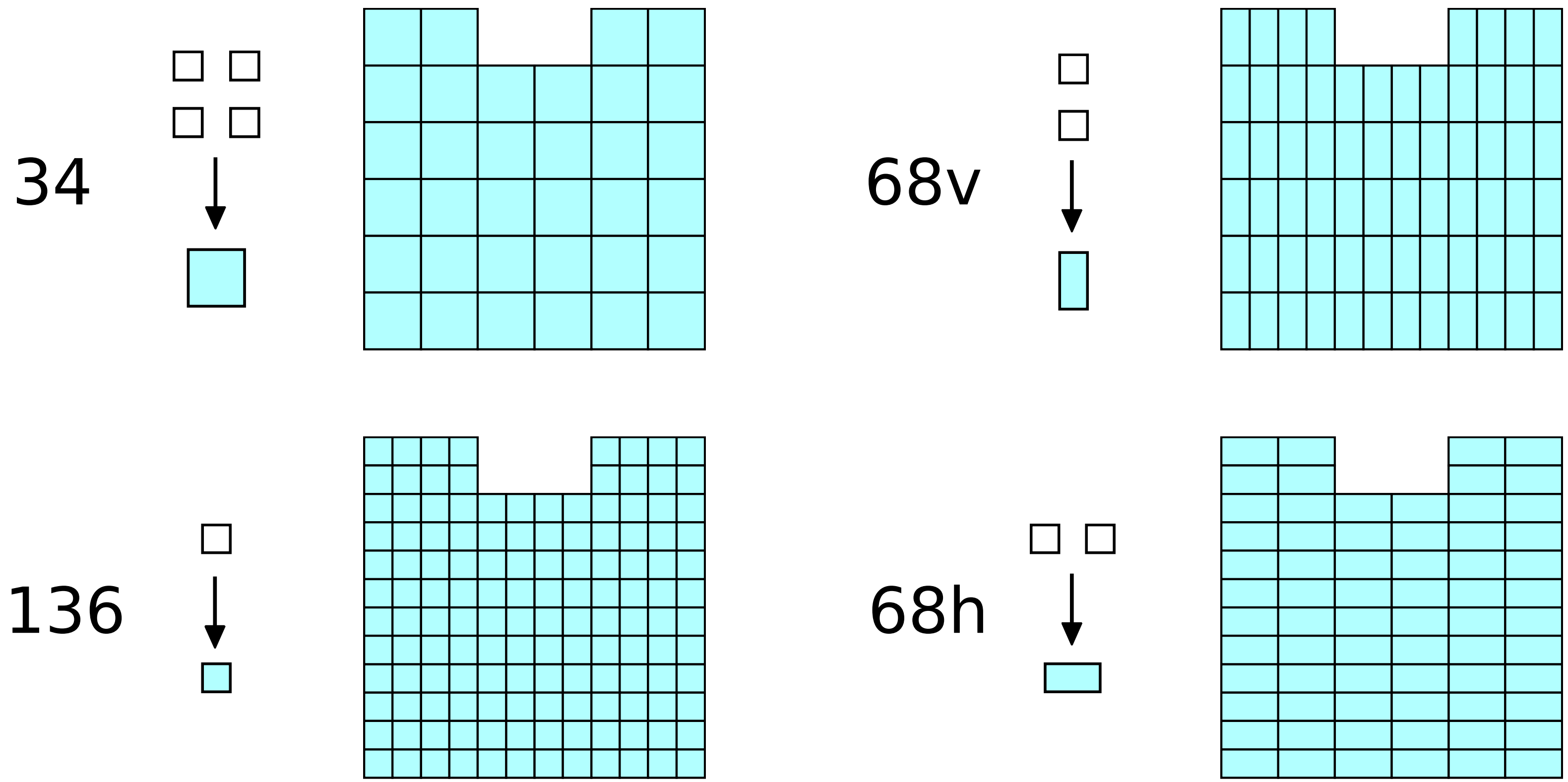
- Indifferent of material model

- Examples of parallel direct unsymmetric solvers:
  - MUMPS (4.10.0)
  - Pardiso (4.12)
  - PaStiX (5.2.1-4030)
  - SuperLU (2.1)
  - WSMP (13.106.10)

Results – memory use, run times

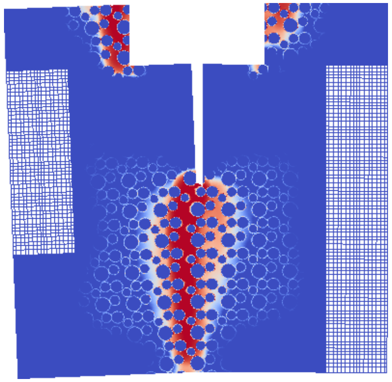


Domain shapes

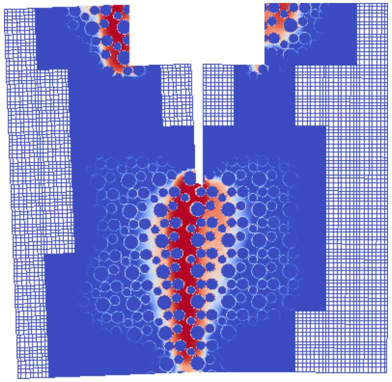


Effect of shape on runtime

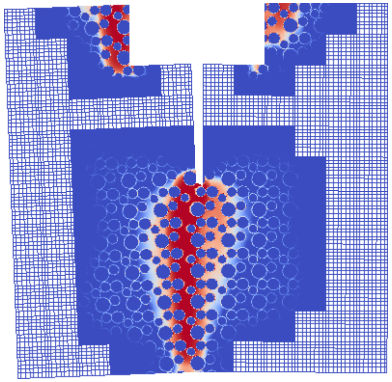
34
4h18m



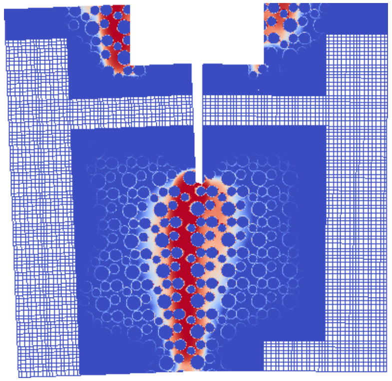
68v
3h45m



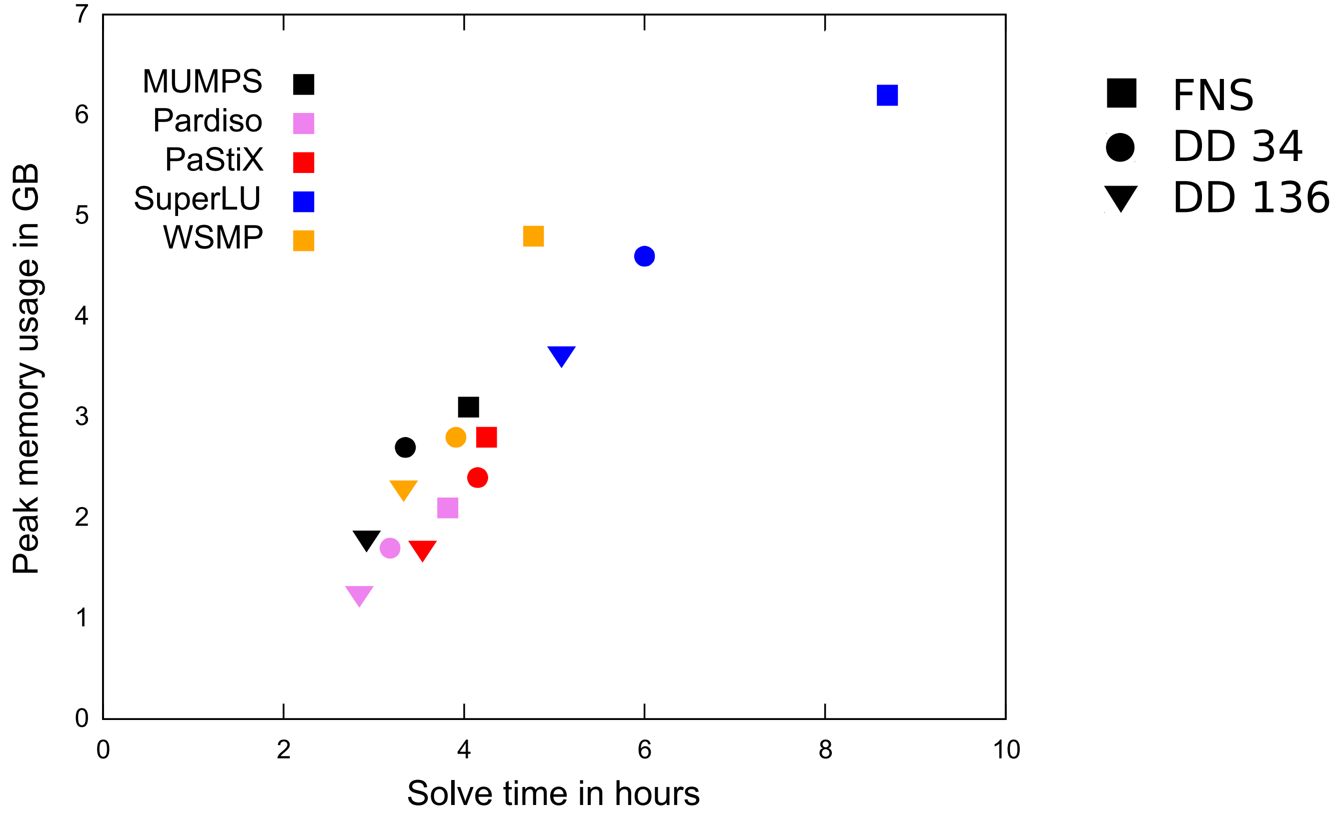
136
3h32m



68h
3h29m



Latest results



Conclusions

- Multiscale framework and parallel direct solver is efficient and robust for damage simulation.
- GEDM needs advanced preconditioners in FETI methods.